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Quasi-particle model for QGP with nonzero densities

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Abstract: We discuss a new quasi-particle model for quark gluon plasma (QGP) with nonzero densities and study the thermodynamics of (2+1)-flavor QGP. Our model with a minimum number of adjustable parameters explains remarkably well the lattice simulation results of Fodor et al. [1].

Keywords: Lattice QCD, Phenomenological Models.

Contents

1. Introduction

Quasi-particle model (qQGP) of quark gluon plasma (QGP) was first proposed by Peshier et al. [2] to explain the non-ideal equation of state (EoS), observed in lattice gauge theory simulations (LGT) [3]. At finite temperature, instead of real quarks and gluons *with* QCD (quantumchromodynamics) *interactions* we may as well consider the system to be made up of *non-interacting* quasi-particles with thermal masses, quasi-quarks and quasi-gluons, and study the thermodynamics. Quasi-particles are quanta of plasma collective modes excited by quarks and gluons through QCD interactions. Initial quasi-particle model was found to be thermodynamically (TD) inconsistent [4] and also not able to fit the more recent LGT results [5]. Gorenstein and Yang reformulated the statistical mechanics (SM) to solve the inconsistency, but end up with an extra undetermined, temperature dependent terms in the expressions for pressure and energy density. This extra term was fixed, forcefully, by applying a constraint relation such that the TD inconsistency term was cancelled. However, as we discuss here [6], above reformulation of SM is not needed and standard SM may be applied to qQGP without any constraints. There is no TD inconsistency in our new qQGP model. Peshier's model with reformulated SM by Gorenstein and Yang has been studied by various groups [7 − 11] with different expressions for thermal masses, effective degrees of freedom, so on. Note that all of above works are based on the reformulation of SM of Gorenstein and Yang. The reason for the TD inconsistency is the wrong choice of pressure and in fact, it must be derived from the partition function as done in text books [12]. A critical comments on these models [2, 4] and our model is discussed in ref. [6] in detail. As we have shown in ref. [6], we may skip this TD inconsistency problem and instead use the original definition of ε and n, and making use of TD relations we may get all TD quantities. As a specific example, here we discuss (2+1)-flavor QGP, studied by Fodor et al. [1] using LGT.

Of course, there are other models like HTL (hard thermal loop) [13], recent FMR (fundamental modular region) gas [14] etc. based on QCD perturbative and non-perturbative calculations, but fails to fit LGT results near to the transition temperature T_c. At the same time phenomenological models of QGP, based on plasma theory with QCD inputs like SCQGP (strongly coupled quark gluon plasma) [15], our present qQGP model seems to fit remarkably well the LGT results with minimum number of parameters. All other qQGP models, field theoretical models [16], Ploykov loop models [17], HTL qQGP models [18] also fit LGT results, but by adjusting 3 or more parameters. Of course, we know that near $T = T_c$, region of phase transition or cross over, it is a low energy phenomena and hence QCD can not be solved by analytical methods like perturbation theory because the coupling constant α_s is not small enough. Probably we need to formulate phenomenological models, just like in the case of hadron spectroscopy, to study TD of QGP near T_C.

2. Phenomenological Model with $\mu = 0$

QGP at thermodynamic equilibrium consists of interacting quarks and gluons which exhibits collective behaviour. Our basic assumption is that this system may be replaced by a system of non-interacting quasi-particles with quantum numbers of quarks and gluons. These quasi-particles have additional thermal masses which are equal to plasma frequencies. Here we differ from other qQGP models where, for example, the thermal mass was taken to be $\sqrt{3/2}$ times the plasma frequency. A general expression for thermal mass or polarization tensor is very complicated expressions which is a function of momentum and frequency. Only at high momentum limit it approaches a simpler form which on further approximations reduces to above form. In view of such a drastic approximation and since we use phenomenological model we may as well take $m_{\rm th} \approx \omega_p$. This is motivated from a similar work in ultra-relativistic (e, e^+, γ) system [19] where they used $m_{\rm th} \approx \omega_p$ and found that the error was less than 3%. In fact with this relation, we get better result than with $m_{\rm th} \approx \sqrt{3/2}\,\omega_p$. Further important point is that the above dispersion relation is obtained using perturbation methods with temperature dependent density distribution function appropriate to ideal system. Then one formulates TD of a non-ideal system. In principle, this must be carried out in a self-consistent manner as discussed in ref. [20], where, for example, density expression is an integral equation since ω_p depends on density. So we need to solve an integral equation self-consistently to get the the density. Here we avoid all above complications and as a phenomenological input, we assume that $m_{\rm th} = \omega_p$.

Following the standard procedure of statistical mechanics [12], the grand partition function is defined as,

$$Q_G = \sum_{s,r} e^{-\beta E_r - \alpha N_s} \ , \tag{2.1}$$

where the sum is over energy states $E_r \equiv \sum_k \epsilon_k\, n_k + E_0$, and particle number states N_s. E_0 is the vacuum energy which we neglect as an approximation without any TD inconsistency, which will be discussed later. ϵ_k and n_k are the single particle energy state and occupation number, respectively. α and β are defined as $\alpha \equiv -\mu/T$ and $\beta \equiv 1/T$ where T and μ are temperature and chemical potential respectively. Next on further simplifications [12], we get,

$$q \equiv \ln Q_G = \mp \sum_{k=0}^{\infty} g_k \ \ln(1 \mp z\, e^{-\beta\epsilon_k}) \ , \tag{2.2}$$

where q is called q-potential and $\mp$ for bosons and fermions. g_k is the degeneracy factor. $z \equiv e^{\mu/T} = e^{-\alpha}$ is called fugacity. ϵ_k is the single particle energy, given by,

$$\epsilon_k = \sqrt{k^2 + m^2} \ ,$$

where k is momentum and m is the total mass which contains both the rest mass and thermal mass ($m_{\rm th}$) of particles. $m_{\rm th}$ may depend on temperature T and chemical potential μ depending on QGP system. With vacuum energy E_0, we may get an additional term $-\beta\, E_0$ for the q-potential. The equation for the q-potential, eq. (2.2), may be further simplified by taking the continuum limit to get,

$$q = \mp \frac{V}{(2\pi)^3} \int d^3k\, g_k \ \ln(1 \mp z e^{-\beta\epsilon_k}) \ . \tag{2.3}$$

Note, that all other quasiparticle QGP models [7, 4, 8 − 11] assume that the pressure is equal to the q-potential, and proceed to evaluate other TD quantities using additional consistency conditions as initiated in ref. [4]. In ref. [6] we proposed instead to define the grand canonical ensemble by the average energy density (ε) and average particle number density (n) and derive the pressure using a standard TD relation. The energy density is defined as,

$$\varepsilon \equiv \frac{< E_r >}{V} = \frac{1}{V} \frac{\sum_{s,r} \ E_r\, e^{-\beta E_r - \alpha N_s}}{Q_G} = -\frac{1}{V} \frac{\partial}{\partial \beta} \ln Q_G = \frac{1}{V} \sum_k \ g_k \ \frac{z\, \epsilon_k e^{-\beta\epsilon_k}}{1 \mp z\, e^{-\beta\epsilon_k}} \ , \tag{2.4}$$

which on continuum limit,

$$\varepsilon = \frac{1}{(2\pi)^3} \int d^3k\, g_k \frac{\epsilon_k}{(z^{-1} e^{\beta\epsilon_k} \mp 1)} \ . \tag{2.5}$$

Similarly, the particle number density is defined as,

$$\begin{aligned} n = \frac{< N >}{V} &\equiv \frac{1}{V} \frac{\sum_{s,r} \ N_s\, e^{-\beta E_r - \alpha N_s}}{Q_G} \\ &= -\frac{1}{V} \frac{\partial}{\partial \alpha} \ln Q_G \\ &= \frac{1}{V}\, z \frac{\partial}{\partial z} \ln Q_G \\ &= \frac{1}{V} \sum_k \ g_k \ \frac{z\, e^{-\beta\epsilon_k}}{1 \mp z\, e^{-\beta\epsilon_k}} \ , \end{aligned} \tag{2.6}$$

which on continuum limit gives,

$$n = \frac{1}{(2\pi)^3} \int d^3k\, g_k \frac{1}{(z^{-1} e^{\beta\epsilon_k} \mp 1)} \ . \tag{2.7}$$

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Hence, using the definition of average energy density and average number density, we have obtained two thermodynamic quantities ε and n from the partition function. The expression for ε, eq. (2.5) may be further simplified as,

$$\varepsilon = \frac{g_f\, T^4}{2\,\pi^2} \sum_{l=1}^{\infty} (\pm 1)^{l-1} z^l \frac{1}{l^4} \left[\left(\frac{m\,l}{T}\right)^3 K_1\left(\frac{m\,l}{T}\right) + 3\left(\frac{m\,l}{T}\right)^2 K_2\left(\frac{m\,l}{T}\right)\right] , \tag{2.8}$$

where g_f is the degeneracy and equal to $g_g \equiv 16$ for gluons and equal to $12\,n_f$ for quarks. n_f is the number of flavors with same mass. K_1 and K_2 are modified Bessel functions of order 1 and 2 respectively. Note that we have neglected the extra $B(T)$ term in the energy density, coming from the vacuum energy E_0. It is a general assumption in quasiparticle models, like Debye's theory of specific heats, liquid helium etc., that the whole thermal energy is used to excite quasiparticles above the vacuum energy at the transition temperature. A similar calculations and comments with the vacuum energy is presented in ref. [6].

Let us now consider our main topic, (2+1)-flavor system, studied by Fodor et al. [1], using our model. It is a QGP with two light (u) and one heavy (s) quarks along with gluons. Let us first consider the case with zero chemical potential and take $z = 1$. Hence we get the energy density, expressed in terms of $e(T) \equiv \varepsilon/\varepsilon_s$, for the quark gluon plasma of quasi-partons is

$$\begin{aligned} e(T) \;=\; & \frac{15}{\pi^4} \frac{1}{(g_f + \frac{21}{2} n_f^{\rm eff})} \sum_{l=1}^{\infty} \frac{1}{l^4} \Bigg(g_f \, \left[\left(\frac{m_g\,l}{T}\right)^3 K_1\left(\frac{m_g\,l}{T}\right) + 3\left(\frac{m_g\,l}{T}\right)^2 K_2\left(\frac{m_g\,l}{T}\right)\right] \\ & +12\, n_u^{\rm eff}\, (-1)^{l-1} \left[\left(\frac{m_u\,l}{T}\right)^3 K_1\left(\frac{m_u\,l}{T}\right) + 3\left(\frac{m_u\,l}{T}\right)^2 K_2\left(\frac{m_u\,l}{T}\right)\right] \\ & +12\, n_s^{\rm eff}\, (-1)^{l-1} \left[\left(\frac{m_s\,l}{T}\right)^3 K_1\left(\frac{m_s\,l}{T}\right) + 3\left(\frac{m_s\,l}{T}\right)^2 K_2\left(\frac{m_s\,l}{T}\right)\right]\Bigg) , \end{aligned} \tag{2.9}$$

where ε_s is the Stefan-Boltzman gas limit of QGP, which may be obtained by taking high temperature limits of eq. (2.8) for gluons and quarks separately and adding them. m_g is the temperature dependent gluon mass ($m_{\rm th}$), which is equal to the plasma frequency, i.e, $m_g^2 = \omega_p^2 = \frac{g^2 T^2}{18}(2N_c + n_f)$. All quarks have both the thermal mass as well as the rest mass and hence, the total mass may be written as

$$m_q^2 = m_{q0}^2 + \sqrt{2}\, m_{q0}\, m_{\rm th} + m_{\rm th}^2 , \tag{2.10}$$

following the idea used in other qQGP models for the system with finite quark masses. Only the difference is that our $m_{\rm th}$ is equal to the plasma frequency due to quarks alone. That is, $m_{\rm th}^2 = \omega_p^2 = \frac{g^2 T^2}{18}\, n_f$. m_{q0} is the rest mass of up or strange quark. g^2 in thermal masses is related to the two-loop order running coupling constant, given by,

$$\alpha_s(T) = \frac{6\pi}{(33 - 2n_f)\ln(T/\Lambda_T)} \left(1 - \frac{3(153 - 19n_f)}{(33 - 2n_f)^2} \frac{\ln(2\ln(T/\Lambda_T))}{\ln(T/\Lambda_T)}\right) , \tag{2.11}$$

where Λ_T is a parameter related to QCD scale parameter. Using thermal masses with above α_s, we can evaluate the $e(T)$ from eq. (2.8). Note that the only temperature dependence in

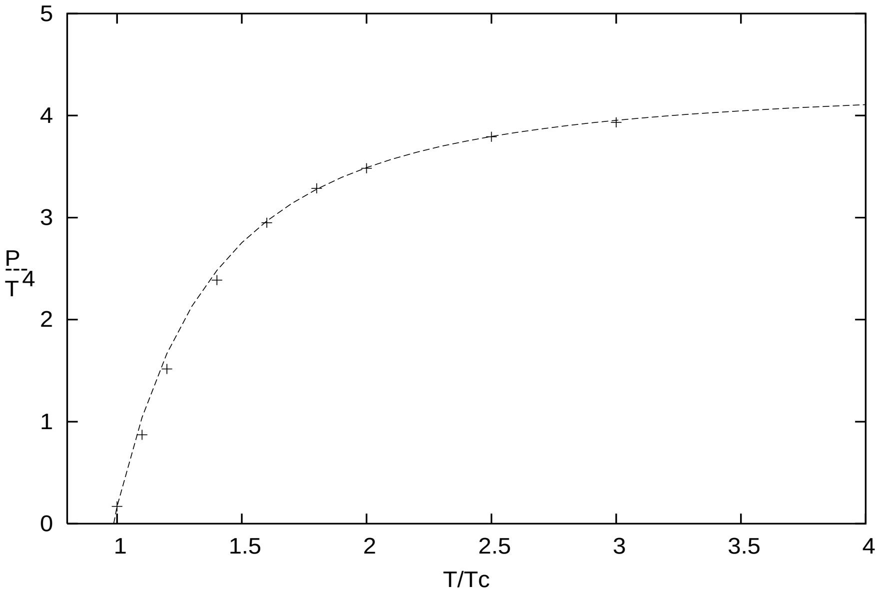

Figure 1: Plots of P/T^4 as a function of T/T_c from our model and lattice results (symbols) [1] for (2+1)-flavor QGP.

$e(T)$ comes from $\alpha_s(T)$, which has the same form as that of lattice simulations [21] with Λ_T as a free parameter. $n_u^{\rm eff}$ and $n_s^{\rm eff}$ are effective number of quarks with up and strange flavor, which differ from 2 and 1, respectively, because of finite masses. However, handling of finite rest masses of quarks are different in LGT studies of Fodor et al. and Bielefeld group [21]. Bielefeld group carried out the simulation with the ratio m_{q0}/T equal to constant and it is straight forward to calculate $n_q^{\rm eff}$, where as Fodor et al. used constant m_{q0} and it is not clear how to calculate $n_q^{\rm eff}$. Hence as done in ref. [22] we take in our calculations $n_u^{\rm eff} = 2$, $n_s^{\rm eff} = 0.5$ and $n_f^{\rm eff} = 2.5$. Similar normalization need to be made to fit the LGT results in other models also, either multiplying LGT data with a factor 1.1 [9] or model's data with .9 [11], or sometime using massive gluon [23] so on. From ε, we may obtain pressure P by using a TD relation $\varepsilon = T\frac{\partial P}{\partial T} - P$ for $\mu = 0$ system and we get

$$\frac{P}{T} = \frac{P_0}{T_0} + \int_{T_0}^{T} dT\, \frac{\varepsilon(T)}{T^2} \ , \tag{2.12}$$

where P_0 and T_0 are pressure and temperature at some reference points. Results are presented in figure 1 along with LGT results. Note that earlier this phenomenological new qQGP model with a single system dependent adjustable parameter explained very well the LGT results of Bielefeld group [21] on QGP system with massless quarks as discussed in ref. [6].

3. Model with finite μ

Let us next consider (2+1)-flavor system with finite μ and LGT results are available for

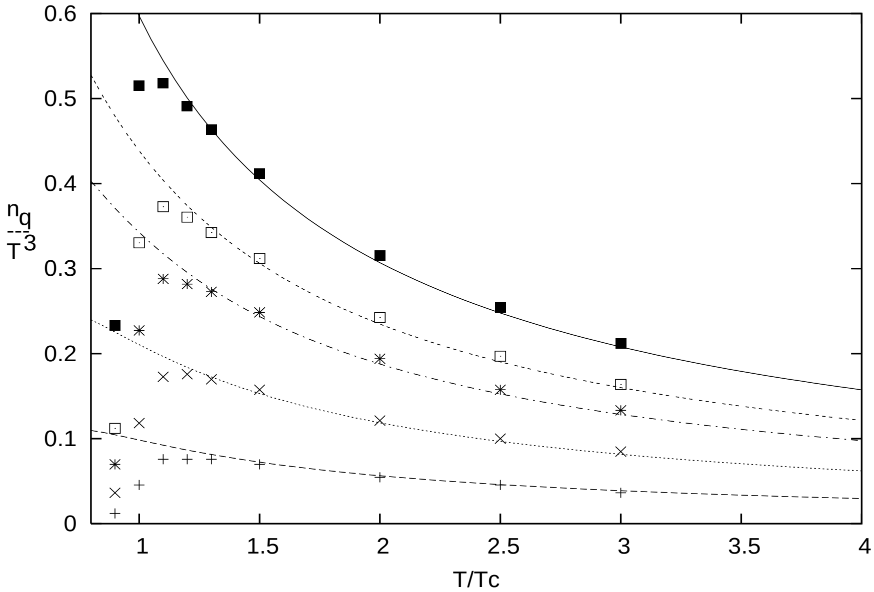

Figure 2: Plots of n_B/T^3 as a function of T/T_c from our model of (2+1)-flavor QGP and also with lattice data (symbols) [1] for $\mu_b = 100$, 210, 330, 410 and 530 MeV from bottom to top.

quark density n_q or baryon density (n_B) and difference in pressure from $\mu = 0$ case ($\Delta P \equiv P(T, \mu) - P(T, \mu = 0)$). Here μ is the quark chemical potential which is one third of the baryon chemical potential. The present LGT results are with μ_B coming from *up* quarks only and hence we need to consider only *up* quark density. Using the expression for the number density, eq. (2.7), and after some algebra, we obtain,

$$\frac{n_q}{T^3} = \frac{12}{\pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} \frac{1}{l^3} \left[\left(\frac{m_q\, l}{T} \right)^2 K_2 \left(\frac{m_q\, l}{T} \right) \sinh \left(\frac{\mu\, l}{T} \right) \right] \ . \tag{3.1}$$

Now we modify earlier $m^2_{\rm th}(T)$ to $m^2_{\rm th}(T, \mu)$ as

$$m^2_{\rm th}(T, \mu) = \frac{g^2 T^2}{18} n_f \left(1 + \frac{\mu}{\pi^2 T^2} \right) \ , \tag{3.2}$$

inspired by QCD perturbative calculations [2]. In our case $n_f = 3$ and g^2 is related to two-loop order running coupling constant, discussed earlier, but need to be modified to take account of finite μ. Following the work of Schneider [24] and Letessier, Rafelski [23], now we change T/Λ_T in eq. (2.11) as

$$\frac{T}{\Lambda_T} \sqrt{1 + a\, \frac{\mu^2}{T^2}} \ , \tag{3.3}$$

where a is equal to $(1.91/2.91)^2$ in the calculation of Schneider and $1/\pi^2$ in a phenomenological model of Letessier and Rafelski. In our model Schneider's $\alpha_s(T, \mu)$ works well.

From n_q, we may obtain other thermodynamic quantities like,

$$\Delta P \equiv P(T,\mu) - P(T,0) = \int_0^{\mu} n_q d\mu \ , \tag{3.4}$$

and so on.

4. Results

In figure 1, we plotted P/T^4 Vs T/T_c for (2+1)-flavor QGP with $\mu = 0$ and compared with LGT data. We took effective number of flavors as $n_u^{\rm eff} = 2$, $n_s^{\rm eff} = .5$ and $n_f^{\rm eff} = 2.5$. Quark rest masses are $m_{u0} = 65\, MeV$ and $m_{s0} = 135\, MeV$, values used in LGT simulations. Only the parameter $t_0 \equiv \Lambda_T/T_c$ is adjusted to get the best fit to LGT data on pressure and is equal to 0.4. In figure 2, the baryon density (n_B/T^3) is plotted as a function of T/T_c for different values of baryon chemical potentials and compared with LGT results. In figure 3, $\Delta P/T^4$ is plotted and compared with LGT results. As can be seen from the figures 2 and 3, curves of n_B/T^3 and $\Delta P/T^4$ almost lie close to that of LGT results without any new parameters.

Thus, using our model with a minimum number of adjustable parameters we can explain the lattice results from the high temperature side up to $1.2\, T_c$. Very near to T_c, probably one need to take account of confinement effects or effects of strongly coupled, non-perturbative system, which is not there in our model. Note that in other qQGP models, they have 4 adjustable parameters and still either n_B/T^3 [11] or P/T^4 [10],one of them, don't fit LGT results well. Result from the QGP liquid model [23], is similar to ours, but with two parameters apart from extra gluon mass.

5. Conclusions

Using our new formulation of qQGP phenomenological model, we were able to explain LGT results on (2+1)-flavor QGP with just a single adjustable parameter which may be related to QCD scale parameter and two fixed parameters ($n_f^{\rm eff}$, P_0). Our formalism is thermodynamically consistent and no need of reformulation of SM with any arbitrary constraints. We start from energy density and number density, well defined in SM for grand canonical ensemble, and develop TD without TD insistency faced by other qQGP models. One more departure from other qQGP models is that we assume that thermal mass is equal to plasma frequency since it arises because of collective effects of plasma, instead of QCD perturbative thermal mass. Earlier, using this model, we explained successfully the LGT results of Bielefeld group on QGP with massless quarks [6]. Hence a simple model of qQGP with a single parameter, related to QCD scale parameter, explains all existing LGT results and may explain future results in LGT and relativistic heavy ion collisions.

Of course, there are many other models which all claim to fit the LGT results since they have more than one adjustable parameters. Models with minimum number of parameters, which fits LGT results well, are SCQGP [15] and liquid model [22, 23], both with two adjustable parameters. All other models involve more than two adjustable parameters.

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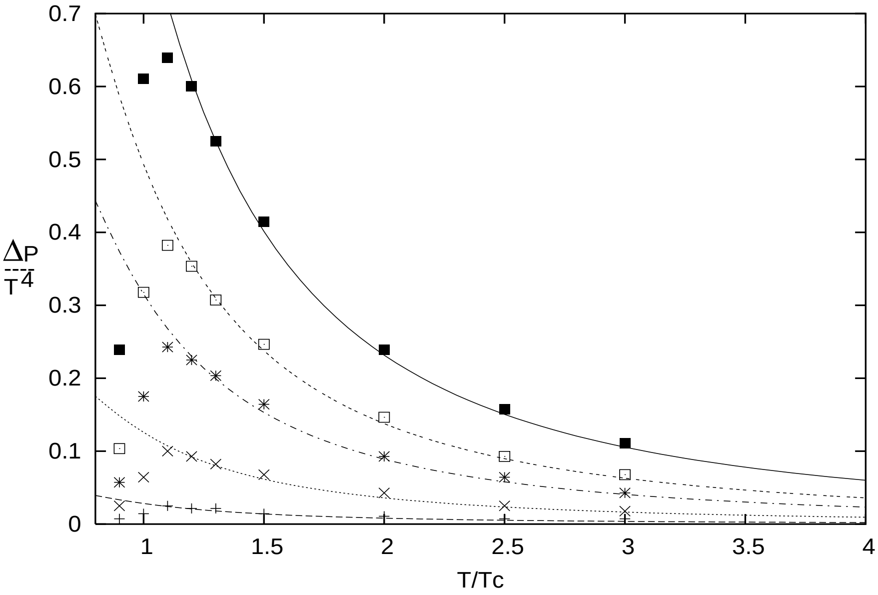

Figure 3: Plots of $\Delta P/T^4$ as a function of T/T_c from our model of (2+1)-flavor QGP and also with lattice data (symbols) [1] for $\mu_b = 100$, 210, 330, 410 and 530 MeV from bottom to top.

Models based on QCD perturbative and non-perturbative calculations [13, 14] fails to fit the LGT results near T_c. Hence, it seems, phenomenological models based on the properties of plasma with QCD inputs explains well the LGT results.

References

[1] Z. Fodor, S.D. Katz and K.K. Szabo, *The QCD equation of state at nonzero densities: lattice result*, *Phys. Lett.* **B 568** (2003) 73 [hep-lat/0208078].

[2] A. Peshier, B. Kampfer, O.P. Pavlenko and G. Soff, *An effective model of the quark-gluon plasma with thermal parton masses*, *Phys. Lett.* **B 337** (1994) 235.

[3] D.H. Rischke, M.I. Gorenstein, A. Schafer, H. Stoecker and W. Greiner, *Nonperturbative effects in the* SU(3) *gluon plasma*, *Phys. Lett.* **B 278** (1992) 19; *Quasiconfinement in the* SU(3) *gluon plasma*, *Z. Physik* **C 56** (1992) 325.

[4] M.I. Gorenstein and S.-N. Yang, *Gluon plasma with a medium dependent dispersion relation*, *Phys. Rev.* **D 52** (1995) 5206.

[5] V.M. Bannur, *Quark gluon plasma as a strongly coupled color-coulombic plasma*, *Eur. Phys. J.* **C 11** (1999) 169 [hep-ph/9811397].

[6] V.M. Bannur, *Comments on quasiparticle models of quark-gluon plasma*, *Phys. Lett.* **B 647** (2007) 271 [hep-ph/0608232]; *Quasi-particle model of QGP — A revisit*, *Eur. Phys. J.* **C 50** (2007) 629 [hep-ph/0508069].

[7] A. Peshier, B. Kampfer, O.P. Pavlenko and G. Soff, *A massive quasiparticle model of the* SU(3) *gluon plasma*, *Phys. Rev.* **D 54** (1996) 2399;

A. Peshier, B. Kampfer and G. Soff, *The equation of state of deconfined matter at finite chemical potential in a quasiparticle description*, *Phys. Rev.* **D 61** (2000) 045203 [hep-ph/9911474].

[8] P. Levai and U.W. Heinz, *Massive gluons and quarks and the equation of state obtained from* SU(3) *lattice QCD*, *Phys. Rev.* **D 57** (1998) 1879 [hep-ph/9710463].

[9] R.A. Schneider and W. Weise, *On the quasiparticle description of lattice QCD thermodynamics*, *Phys. Rev.* **C 64** (2001) 055201 [hep-ph/0105242].

[10] K.K. Szabo and A.I. Toth, *Quasiparticle description of the QCD plasma, comparison with lattice results at finite t and μ*, *JHEP* **06** (2003) 008 [hep-ph/0302255].

[11] Y.B. Ivanov, V.V. Skokov and V.D. Toneev, *Equation of state of deconfined matter within dynamical quasiparticle description*, *Phys. Rev.* **D 71** (2005) 014005 [hep-ph/0410127].

[12] R.K. Pathria, *Statistical mechanics*, Butterworth Heinemann, Oxford (1997).

[13] J.P. Blaizot, E. Iancu and A. Rebhan, *The entropy of the QCD plasma*, *Phys. Rev. Lett.* **83** (1999) 2906 [hep-ph/9906340].

[14] D. Zwanziger, *Equation of state of gluon plasma from fundamental modular region*, *Phys. Rev. Lett.* **94** (2005) 182301 [hep-ph/0407103].

[15] V.M. Bannur, *Strongly coupled quark gluon plasma (SCQGP)*, *J. Phys.* **G 32** (2006) 993 [hep-ph/0504072].

[16] C. Ratti, M.A. Thaler and W. Weise, *Phases of QCD: lattice thermodynamics and a field theoretical model*, *Phys. Rev.* **D 73** (2006) 014019 [hep-ph/0506234].

[17] D.H. Rischke, *The quark-gluon plasma in equilibrium*, *Prog. Part. Nucl. Phys.* **52** (2004) 197.

[18] A. Rebhan and P. Romatschke, *HTL quasiparticle models of deconfined QCD at finite chemical potential*, *Phys. Rev.* **D 68** (2003) 025022 [hep-ph/0304294].

[19] M.V. Medvedev, *Thermodynamics of photons in relativistic e^+e^- gamma plasmas*, *Phys. Rev.* **E 59** (1999) 4766 [astro-ph/9902235].

[20] V.M. Bannur, *Self-consistent quasi-particle model for relativistic plasma*, *Phys. Rev.* **E 73** (2006) 067401 [hep-ph/0602222].

[21] G. Boyd et al., *Equation of state for the* SU(3) *gauge theory*, *Phys. Rev. Lett.* **75** (1995) 4169 [hep-lat/9506025]; *Thermodynamics of* SU(3) *lattice gauge theory*, *Nucl. Phys.* **B 469** (1996) 419 [hep-lat/9602007];
E. Laermann and O. Philipsen, *The status of lattice QCD at finite temperature*, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 163 [hep-ph/0303042];
F. Karsch, E. Laermann and A. Peikert, *The pressure in* $2, 2+1$ *and* 3 *flavour QCD*, *Phys. Lett.* **B 478** (2000) 447 [hep-lat/0002003].

[22] S. Hamieh, J. Letessier and J. Rafelski, *Exploding quark-gluon plasma fireball*, *Phys. Rev.* **C 62** (2000) 064901 [hep-ph/0006085].

[23] J. Letessier and J. Rafelski, *QCD equations of state and the QGP liquid model*, *Phys. Rev.* **C 67** (2003) 031902 [hep-ph/0301099].

[24] R.A. Schneider, *The QCD running coupling at finite temperature and density*, hep-ph/0303104.